Chapter 1 discussion.

Wednesday, May 19, 2021 1:57 PM

$\Omega = \mathbb{N}$

$\mathcal{A}_n = \{\emptyset, \mathbb{N}, \{n\}, \mathbb{N} \setminus \{n\}\}$

$\bigcup \mathcal{A}_n = \{\emptyset, \mathbb{N}, \{j\}, \mathbb{N} \setminus \{j\}, j \in \mathbb{N}\}$

$\{1\} \cup \{2\} \notin$

$\mathcal{B}_j = \{A \subset \{1, 2, \dots, j\}$ or $A^c \subset \{1, 2, \dots, j\}\}$

$\mathcal{B}_j \subset \mathcal{B}_{j+1}$

$\mathcal{B} = \bigcup \mathcal{B}_j$. $\quad \underbrace{\{1\} \cup \{3\} \cup \{5\} \cup \dots}$ = odd numbers $\notin \mathcal{B}$, $\cancel{\mathcal{A}}$ $\cancel{\mathcal{A}}$

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6 $\quad 0 < a_n \quad b_n > 1 \quad \underbrace{a_n \to 0} \quad \underbrace{b_n \to 1}$

$a_n \le b_n$

$A_n = \{x : a_n \le x \le b_n\}$

$\underbrace{\limsup A_n}_{\bigcap \bigcup \text{ i.o.}} = \underbrace{\liminf A_n}_{\bigcup \bigcap \text{ all but finitely many}} = (0, 1]$

$x \le 0 \Rightarrow x < a_n \Rightarrow x \notin A_n \quad \forall n$

$x > 1 \Rightarrow \exists N : \underbrace{n > N} : b_n < x. \Rightarrow x \notin A_n$

$0 < x \le 1 \quad \exists N_1 : \underbrace{n > N_1} \quad a_n < x$

$\forall n : \quad x < b_n. \quad \in \liminf A_n$

$$\left.\begin{array}{l} (0, 1] \subset \liminf A_n. \\ (0, 1]^c \cap \limsup A_n = \emptyset \end{array}\right) \Rightarrow (0, 1] = \dots$$

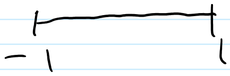
(33) $\mathcal{A} = \{[-\infty, x]\} = \{\{-\infty\} \cup (-\infty, x]\}$.

$\sigma(\mathcal{A}) = \sigma(\mathcal{B})$.

$B \in \mathcal{B} \Rightarrow B \in \sigma(\mathcal{A})$

$A \in \mathcal{A} \Rightarrow A \in \sigma(\mathcal{B})$

$$\varphi(x) = \frac{x}{1-|x|}.$$



$\varphi(-1) = -\infty$

$\varphi(1) = \infty$

$\mathcal{B}([-1,1]) = \sigma(\underbrace{[-1, x]})$

$$\varphi\left(\mathcal{B}([-1,1])\right) = \sigma\left([\underbrace{\varphi(-1)}_{-\infty}, \underbrace{\varphi(x)}_{y}]\right) = \mathcal{B}([-\infty,\infty])$$

bijection
increasing

$\mathbb{N} = \Omega$.

$\mathcal{P} = \{\underbrace{\emptyset, \{j\}, j \in \mathbb{N}}\}$ — P system. $\mathcal{M} = \mathcal{P}$.

$\sigma(\mathcal{P}) = 2^{\mathbb{N}}$

$\mathcal{M} = \{\underbrace{\{j\}}, \underbrace{\{l : l \geq j\}}\} \neq 2^{\mathbb{N}}$.